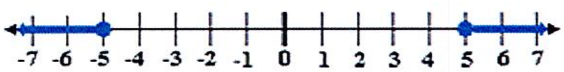
Mr. Michael T. Davis Pre-Calculus	Unit 1 (1.1-1.9) – Practice Test October 19, 2016

Name: Solutions

1.1 INTERVAL NOTATION

1) Write the domain of each graph in both **inequality** and **interval** notation.

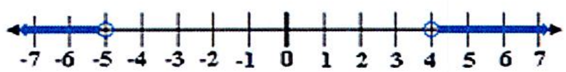
Graph 1



Inequality: $x \le -5$ or $x \ge 5$

Interval: $(-\infty,-5] \cup [5,\infty)$

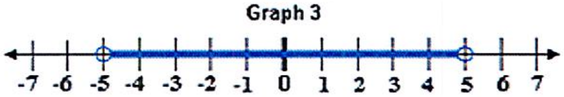
Graph 2



Inequality: $x < -5$ or $x > 4$

Interval: $(-\infty,-5) \cup (4,\infty)$

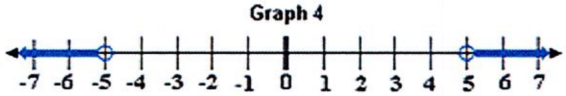
Graph 3



Inequality: $-5 < x < 5$

Interval: $(-5,5)$

Graph 4



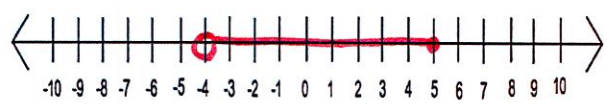
Inequality: $x < -5$ or $x > 5$

Interval: $(-\infty,-5) \cup (5,\infty)$

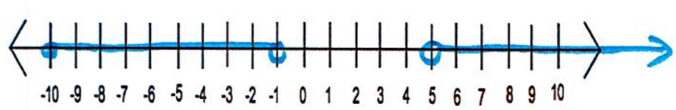
2) Graph each interval on a number line

a. $[-3,\infty)$

b. $(-4,5]$



c. $[-10,-1) \cup (5,\infty)$



3) Write each inequality in interval notation.

a. $-9 < x \le 6$ $(-9, 6]$

b. $-5 \le x < 4$ *or* $x > 8$ $[-5, 4) \cup (8, \infty)$

c $x < -5$ *or* $x \ge 0$ $(-\infty, -5) \cup [0, \infty)$

4) Solve each inequality. Graph the solution set on a number line, AND write the solution in interval notation.

a. $5 - \frac{1}{3}n \le 6$

$-\frac{1}{3}n \le 1$

$n \ge -3$

$[-3, \infty)$

b. $-8 \le \frac{3}{4}p - 2 < 10$

$-6 \le \frac{3}{4}p < 12$

$-8 \le p < 16$

$[-8, 16)$

c. $3x - 8 > 1$ *or* $2x + 9 < -3$

$3x > 9$ $2x < -12$

$x > 3$ $x < -6$

$(-\infty, -6) \cup (3, \infty)$

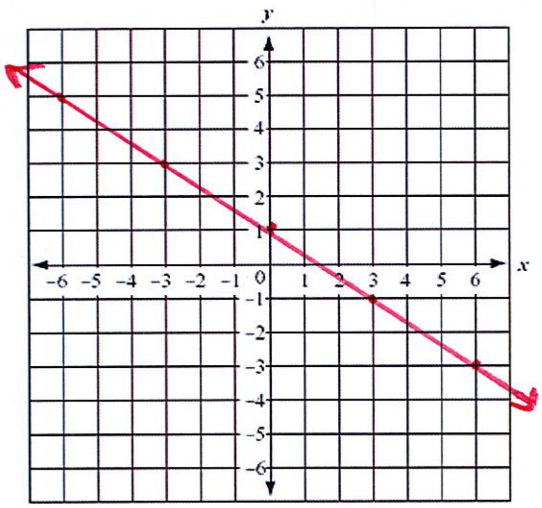
1.2 LINEAR FUNCTIONS

1. A lake near the Arctic Circle is covered by a 2-meter thick sheet of ice during the cold winter months. When spring arrives, the warm air gradually melts the ice, causing its thickness to decrease at a constant rate. After 3 weeks, the sheet of ice is only 1.25 meters thick. Let y represent the thickness of the ice, and let x represent time in weeks. Write a linear function that gives the thickness of the ice as a function of time. $(x, y) = (\text{weeks}, \text{inches})$

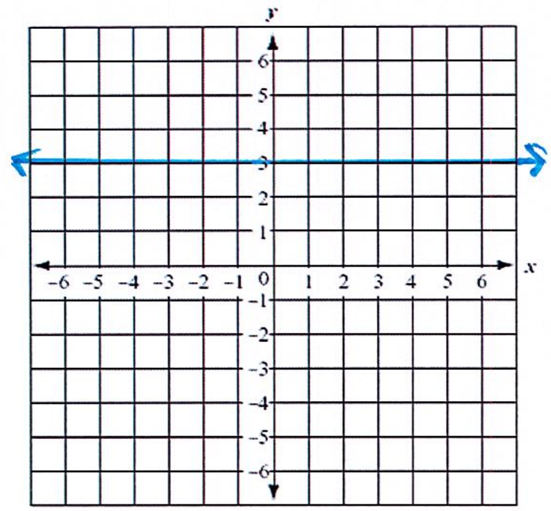
$(0, 2)$ & $(3, 1.25)$

$m = \frac{2 - 1.25}{0 - 3} = \frac{-0.75}{3} = -0.25$ $y = -0.25x + 2$

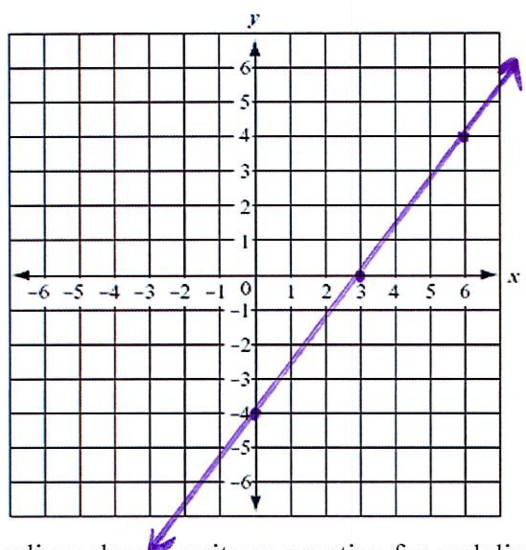
2. Graph the line with equation $y = \frac{-2}{3}x + 1$



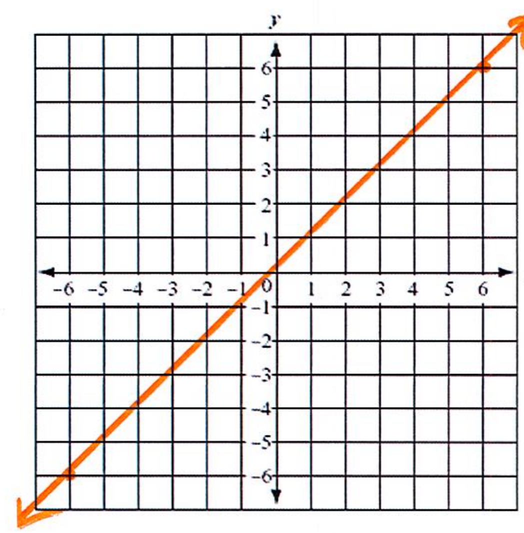
3. Graph the line with equation $y = 3$



4. Graph the line with equation $4x - 3y = 12$

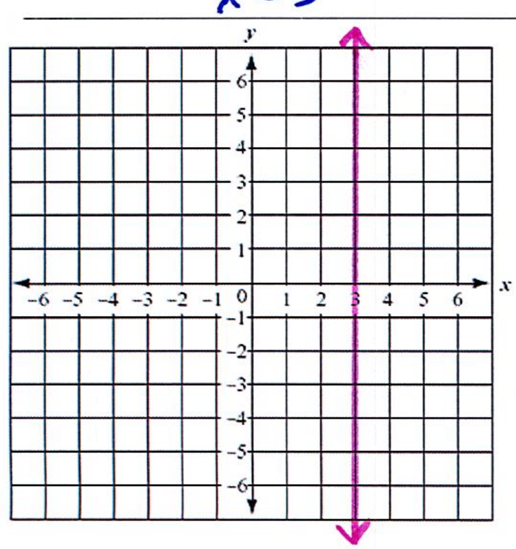


5. Graph the line with equat. $y = x$

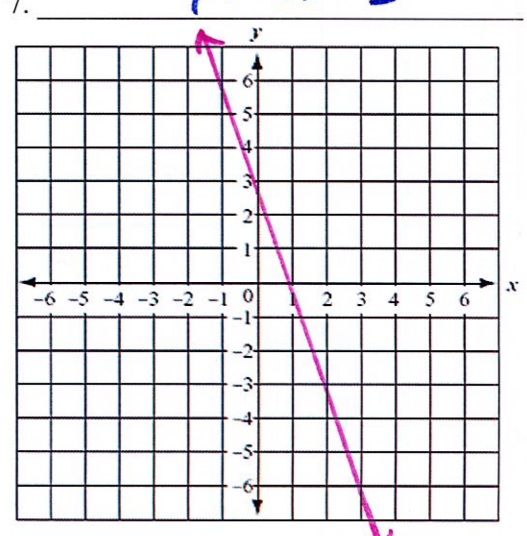


Given the lines shown, write an equation for each line in any form.

6. $x = 3$



7. $y = -3x + 3$



8. Given a line containing the points $(-3,3)$ and $(9,7)$. Find an equation of the line in slope-intercept form. Convert your equation to standard from.

$m = \frac{7-3}{9--3} = \frac{4}{12} = \frac{1}{3}$

$y = \frac{1}{3}x + b$

$(9,7)$ $7 = \frac{1}{3}(9) + b$

$7 = 3 + b$

$4 = b$

$y = \frac{1}{3}x + 4$

$3y = x + 12$

$-x + 3y = 12$

9. Given a line containing the points $(4,7)$ and $(-1,7)$. Find an equation of the line.

$m = \frac{7-7}{4--1} = \frac{0}{5} = 0$ horizontal line

Equation is $y = 7$

10. Given a line containing the points $(3,5)$ and $(3,1)$. Find an equation of the line.

$m = \frac{5-1}{3-3} = \frac{4}{0} =$ undefined vertical line

Equation is $x = 3$

11. The two given equations represent lines. Are the lines parallel or Perpendicular or neither? Explain briefly why. $x + 2y = 5$ & $2x - y = 4$

$2y = -x + 5$ $\quad$ $-y = -2x + 4$

$y = -\frac{1}{2}x + \frac{5}{2}$ $\quad$ $y = 2x - 4$

The slopes are $m = -\frac{1}{2}$ and $m = 2$ which are opposite reciprocals

Therefore the lines are $\perp$

12. Line A has equation $y = 3x + 1$. Line B contains the point $(-1,-8)$ and is parallel to line A. Determine an equation for line B.

Line A has slope $m = 3$. Line B has slope $m = 3$.

Using $(-1,-8)$ & $m = 3$, the equation is

$y + 8 = 3(x + 1)$

13. Line A has equation $y = \frac{2}{5}x - 6$. Line B contains the point $(4,-9)$ and is perpendicular to line A. Determine an equation for line B.

Line A has slope $m = \frac{2}{5}$. Line B has slope $m = \frac{-5}{2}$

Using $(4,-9)$ & $m = \frac{-5}{2}$, $y + 9 = \frac{-5}{2}(x - 4)$

1.3 BASES & EXPONENTS

Simplify each expression (do not leave any negative exponents):

1. $\sqrt[3]{64} =$ 4

2. $\sqrt[6]{64} =$ 2

3. $9^{\frac{3}{2}} =$ $\left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$

4. $27^{\frac{4}{3}} =$ $\left(27^{\frac{1}{3}}\right)^4 = 3^4 = 81$

5. $\dfrac{7^5}{7^7} =$ $\dfrac{1}{7^2} = \dfrac{1}{49}$

6. $\dfrac{x^{14}}{x^6} =$ $= x^8$

7. $\dfrac{35x^{10}y}{5x^6y^3} =$ $\dfrac{7x^4}{y^2}$

8. $\dfrac{x^{-2}}{x^3} =$ $\dfrac{1}{x^2x^3} = \dfrac{1}{x^5}$

9. $2x^2y^4 + 8x^2y^4 =$ $10x^2y^4$

10. $\left(2x^2y^3\right)^4 + \left(3x^4y^6\right)^2 =$
$16x^8y^{12} + 9x^8y^{12}$
$25x^8y^{12}$

11. $\dfrac{(x+y)^3}{(x+y)^2} =$ $x+y$

12. $\left(\dfrac{3m^2n^7}{m}\right)^0 =$ 1

1.4 MULTIPLYING, FACTORING & SOLVING POLYNOMIALS

1. Multiply $(2x-5)^2$ $= (2x-5)(2x-5) = 4x^2 - 10x - 10x + 25$
$= 4x^2 - 20x + 25$

2. Multiply $-3(x-4)(x+4)$ $= -3(x^2-16) = -3x^2 + 48$

3. Multiply $(2x+5)(4x-3)$ $= 8x^2 - 6x + 20x - 15 = 8x^2 + 14x - 15$

4. Factor $x^2 - 9x$ $= x(x-9)$

5. Factor $3x^2 + 14x + 8$ $= (3x+2)(x+4)$

6. Factor $8x^2 + 14x - 15$ $= (2x+5)(4x-3)$

1.5 QUADRATIC FUNCTIONS – CONVERTING FORMS

1. Convert the quadratic equation $f(x) = x^2 + 6x + 10$ to vertex form

$f(x) = (x^2 + 6x + 9) + 10 - 9 = (x+3)(x+3) + 1$

$f(x) = (x+3)^2 + 1$ $V(-3,1)$

2. Convert the quadratic function $f(x) = \frac{1}{2}(x+4)^2 + 3$ to standard form

$f(x) = \frac{1}{2}(x^2 + 8x + 16) + 3 = \frac{1}{2}x^2 + 4x + 8 + 3$

$f(x) = \frac{1}{2}x^2 + 4x + 11$

3. Convert the quadratic function $g(x) = 2x^2 - 5x - 3$ to factored form

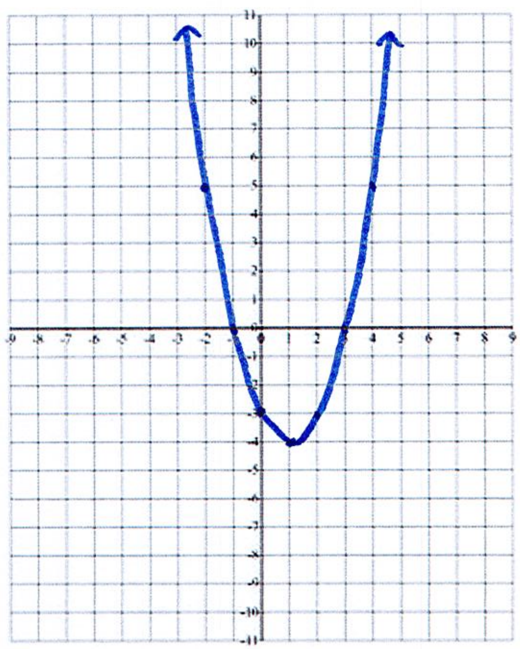
$g(x) = (2x+1)(x-3)$

1.6 QUADRATIC FUNCTIONS – FINDING ZEROS & GRAPHING

1. Graph $f(x) = (x-1)^2 - 4$

$V(1,-4)$

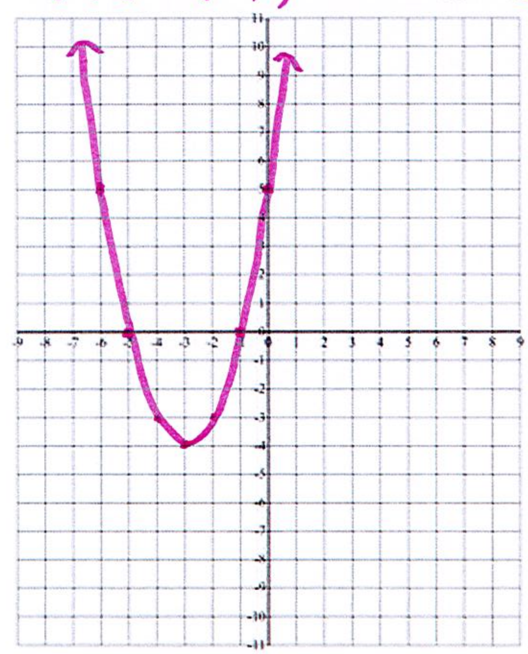
$a = 1$



2. Graph $k(x) = x^2 + 6x + 5 = (x+1)(x+5)$

$a = 1$

$(-1,0)$ & $(-5,0)$ axis $x = -3$

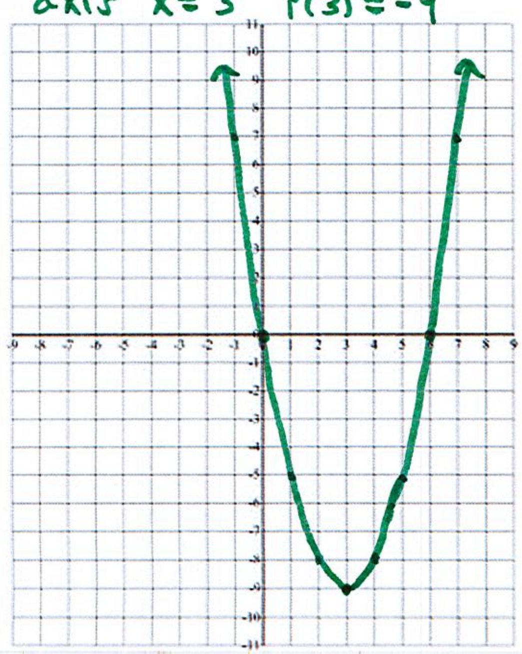


$f(-3) = (-3)^2 + 6(-3) + 5 = 9 - 18 + 5 = -4$

3 Graph $p(x) = x(x-6)$ $(0,0)$ & $(6,0)$

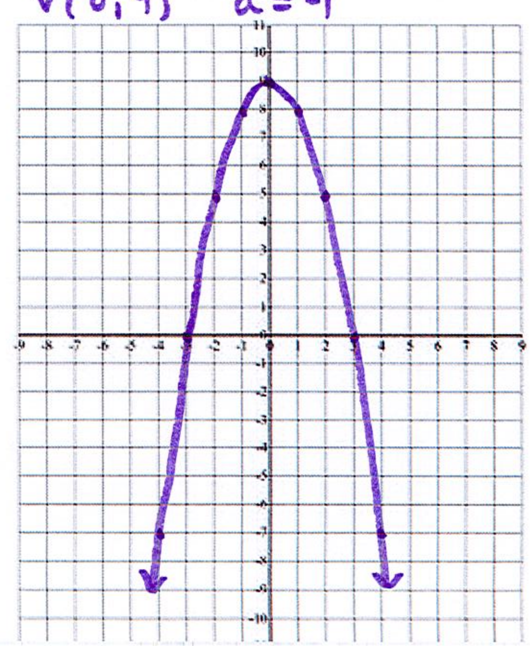
axis $x = 3$ $p(3) = -9$

$a = 1$



4. Graph $g(x) = 9 - x^2 = (3-x)(3+x)$

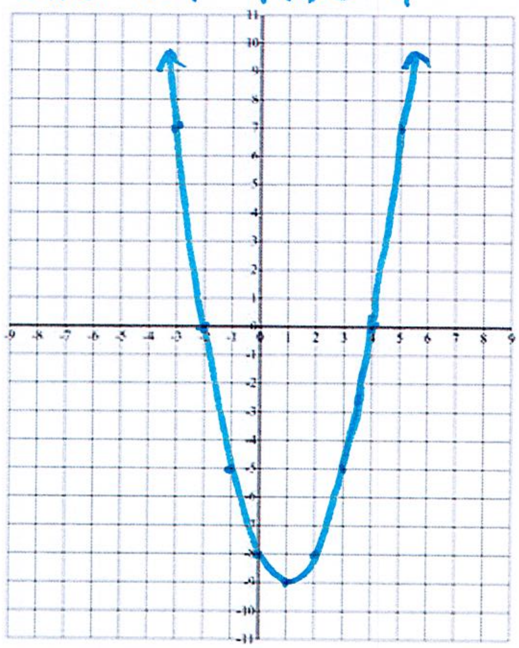
$V(0,9)$ $a = -1$



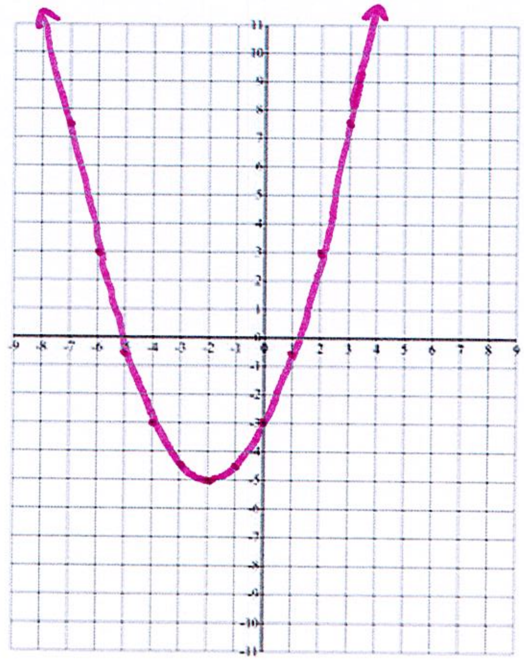
$a=1$

5. $p(x)=(x-4)(x+2)$ $(4,0),(-2,0)$

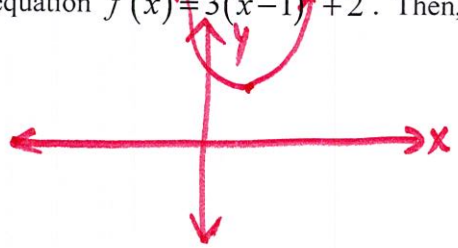
axis $x=1$ $p(1)=-9$



6. Write an equation for the parabola shown



7. Without graphing, determine the number of real zeros (x-intercepts) of the parabola with equation $f(x)=3(x-1)^2+2$. Then, find the real zeros, i.e. $0=3(x-1)^2+2$



$V(1,2)$

No real zeros

8. Without graphing, determine the number of real zeros (x-intercepts) of the parabola with equation $f(x)=\frac{1}{4}(x+1)^2-3$. Then, find the real zeros, i.e. $0=\frac{1}{4}(x+1)^2-3$

$0=\frac{1}{4}(x+1)^2-3$

$3=\frac{1}{4}(x+1)^2$

$12=(x+1)^2$

$\pm\sqrt{12}=x+1$

$-1\pm\sqrt{4\cdot 3}=x$

$-1\pm 2\sqrt{3}=x$

$\{-1+2\sqrt{3}, -1-2\sqrt{3}\}$

9. Determine the real zeros (x-intercepts) of the parabola with equation $h(x)=(3x+7)(2x-5)$

$3x+7=0$

$3x=-7$

$x=\frac{-7}{3}$

$2x-5=0$

$2x=5$

$x=\frac{5}{2}$

$\{\frac{-7}{3}, \frac{5}{2}\}$

10. Determine the real zeros (x-intercepts) of the parabola with equation $f(x)=3x^2-18x$

$0 = 3x^2 - 18x$

$0 = 3x(x-6)$

$3x = 0 \quad x-6=0$

$x=0 \quad x=6$

$\{0, 6\}$

11. Determine the real zeros (x-intercepts) of the parabola with equation $p(x)=x^2-5x-14$.

$0 = (x-7)(x+2)$

$\{7, -2\}$

12. Determine the real zeros (x-intercepts) of the parabola with equation $k(x)=2x^2-50$

$0 = 2x^2 - 50$

$0 = x^2 - 25$

$0 = (x-5)(x+5)$

$\{5, -5\}$

13. Determine the real zeros (x-intercepts) of the parabola with equation $g(x)=x^2+2x-6$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-6)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{4+24}}{2}$

$x = \dfrac{-2 \pm \sqrt{28}}{2}$

$x = \dfrac{-2 \pm \sqrt{4 \cdot 7}}{2}$

$x = \dfrac{-2 \pm 2\sqrt{7}}{2}$

DNF

$x = -1 \pm \sqrt{7}$

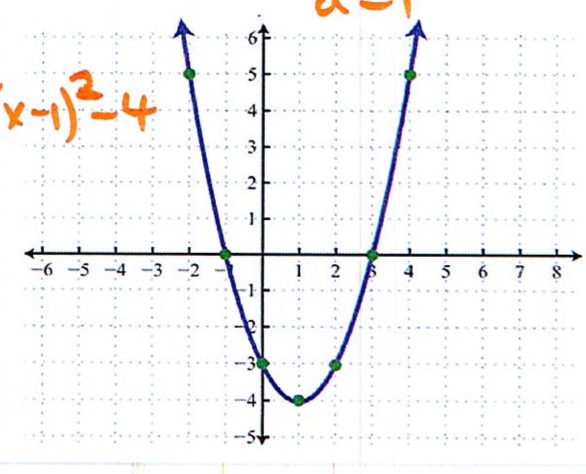
$\left\{ \begin{matrix} -1+\sqrt{7} \\ -1-\sqrt{7} \end{matrix} \right\}$

14. For each parabola shown, write a quadratic equation in the form of your choice.

a.

$a = 1$

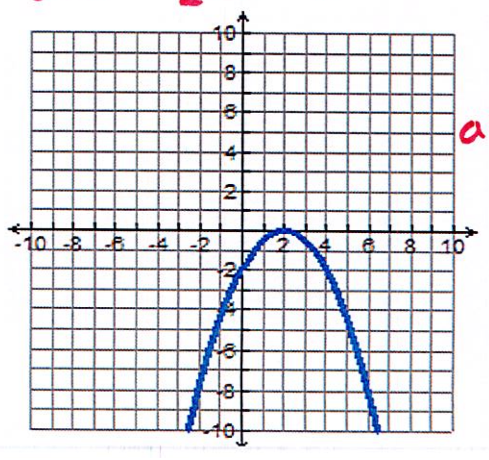
$f(x) = (x-1)^2 - 4$



b.

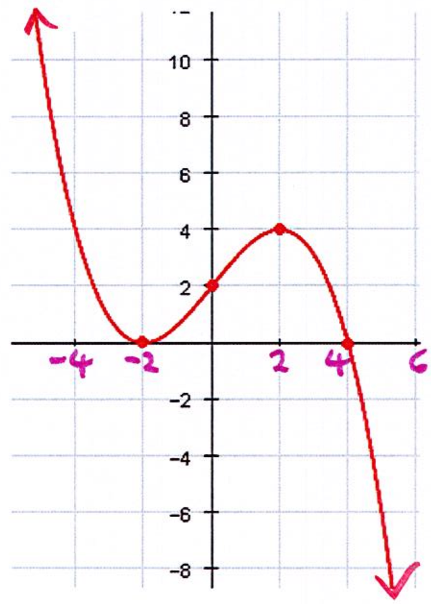
$g(x) = -\frac{1}{2}(x-2)^2$

$a = -\frac{1}{2}$



1.7 FUNCTION GRAPH ANALYSIS

I. Shown is the graph of $y = g(x)$



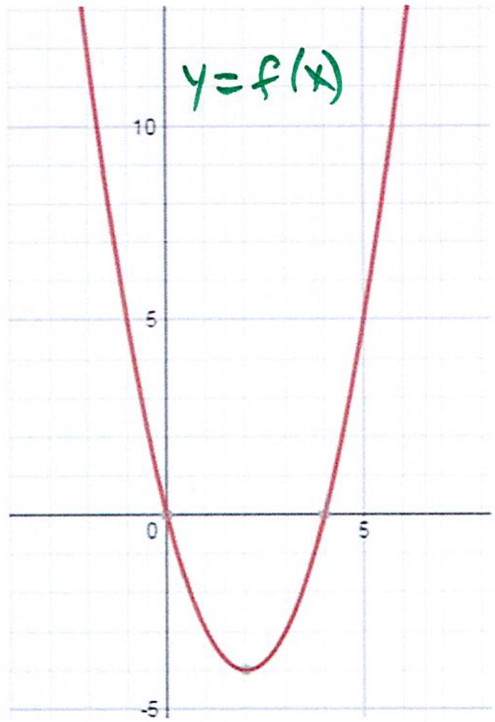
1. Evaluate: a. $g(0) =$ 2 b. $g(2) =$ 4
2. Write the domain and range of the function using interval notation. D: $(-\infty, \infty)$ R: $(-\infty, \infty)$
3. State the interval(s) on which the function is: a. increasing b. decreasing $[-2, 2]$ $(-\infty, -2] \cup [2, \infty)$
4. State the interval(s) for which: a. $g(x) > 0$ b. $g(x) < 0$ $(-\infty, 2) \cup (2, 4)$ $(4, \infty)$
5. State each value: a. the maximum value of $y = g(x)$ b. the minimum value of $y = g(x)$ None None
6. Solve $g(x) = 2$, i.e. for what value(s) of x does $g(x) = 2$ hold true? $x = -3.5$, $x = 0$, $x = 3.5$
7. State the coordinates of each (approximate if necessary): a. any x-intercepts b. the y-intercept $(-2, 0)$ & $(4, 0)$ $(0, 2)$

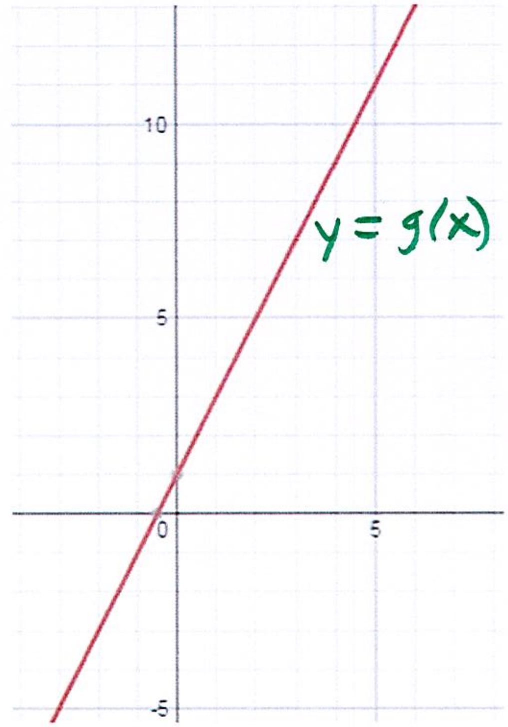
1.8 OPERATING ON FUNCTIONS & COMPOSITE FUNCTIONS

Given the two functions $f(x)=x^2-5$ **and** $g(x)=x-3$**:**

1. Evaluate $(f+g)(x)$ $x^2-5+x-3$ x^2+x-8	2. Evaluate $f(4)-g(4)$ $4^2-5-(4-3)$ $16-5-(1)$ $11-1$ 10
3. Evaluate $f(x)\cdot g(x)$ $(x^2-5)(x-3)$ $x^3-3x^2-5x+15$	4. Evaluate $\left(\frac{f}{g}\right)(-1)$ $\frac{(-1)^2-5}{-1-3}=\frac{1-5}{-4}=\frac{-4}{-4}=1$
5. Evaluate $f(g(x))$ $f(x-3)$ $(x-3)^2-5$ $x^2-6x+9-5$ x^2-6x+4	6. Evaluate $(g\circ f)(1)$ $= g(f(1))$ $g((1)^2-5)=g(-4)=-4-3=-7$
7. State the domain and range of $f(x)$ D: $(-\infty,\infty)$ R: $[-5,\infty)$	8. State the domain and range of $g(f(x))$ $g(x^2-5)=x^2-5-3=x^2-8$ D: $(-\infty,\infty)$ R: $[-8,\infty)$

Use the graphs of $y=f(x)$ and $y=g(x)$ shown below to answer the questions ~~23~~ 9 through ~~32~~ 16

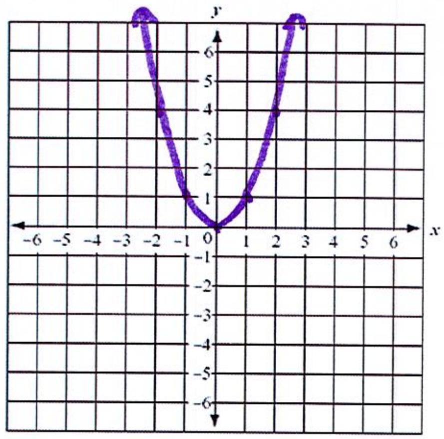




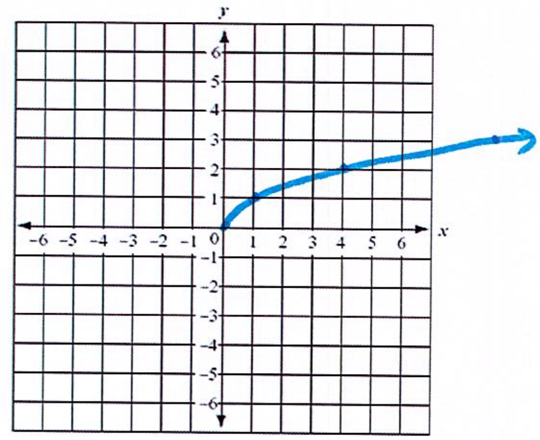
9. Evaluate $(f+g)(0)$ $f(0)+g(0)=0+1=1$	10. Evaluate $(g\circ f)(x) = g(f(x))$ cannot do without a value of x
11. Evaluate $f(-1)-g(-1)$ $5-(-1)$ $5+1$ 6	12. Evaluate $f(g(1)) = f(3) = -3$
13. Evaluate $f(-2)\cdot g(-2)$ $(12)(-3)$ -36	14. Evaluate huh?
15. Evaluate $\left(\frac{f}{g}\right)(5) = \frac{f(5)}{g(5)} = \frac{5}{11}$	16. Evaluate $\left(\frac{f}{g}\right)(-3) = \frac{f(-3)}{g(-3)} = \frac{21}{-5} = \frac{-21}{5}$

1.9 PARENT FUNCTIONS & TRANSFORMATIONS

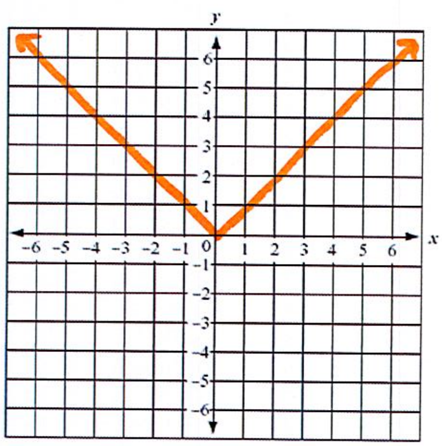
1. Graph $f(x)=x^2$



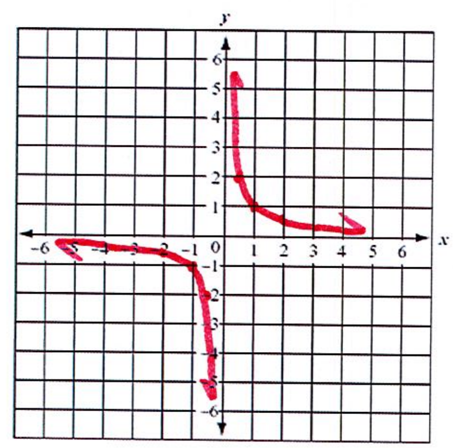
2. Graph $f(x)=\sqrt{x}$ or $f(x)=x^{\frac{1}{2}}$



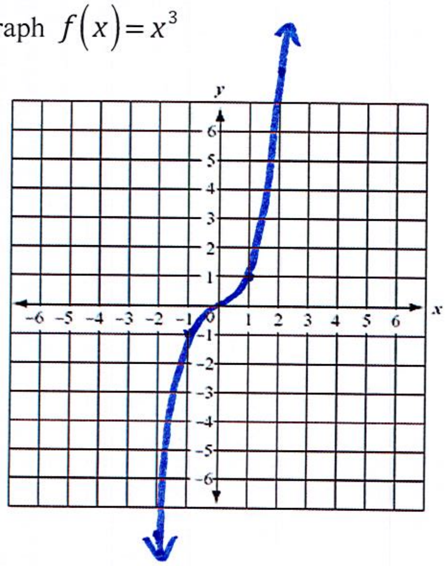
3. Graph $f(x)=|x|$



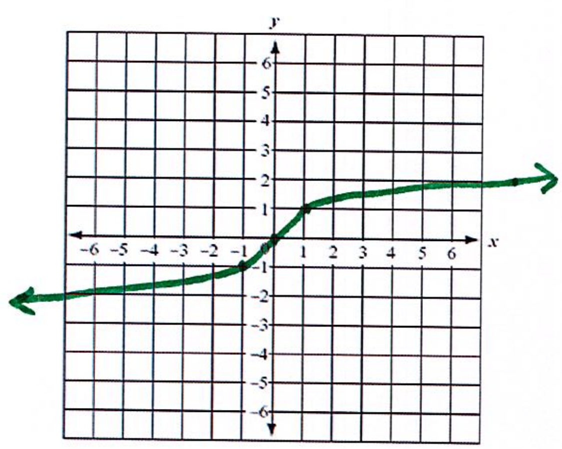
4. Graph $f(x)=\frac{1}{x}$



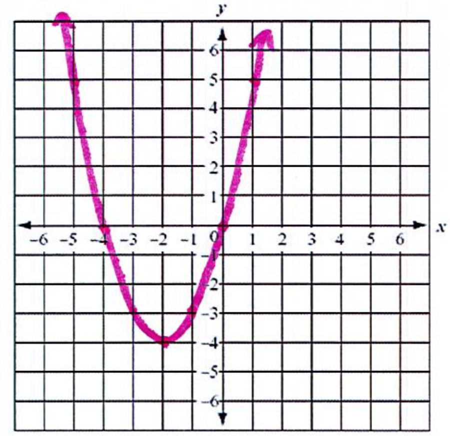
5. Graph $f(x)=x^3$



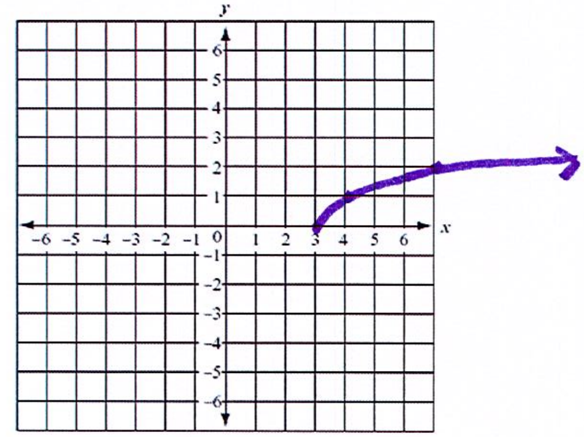
6. Graph $f(x)=x^{\frac{1}{3}}$



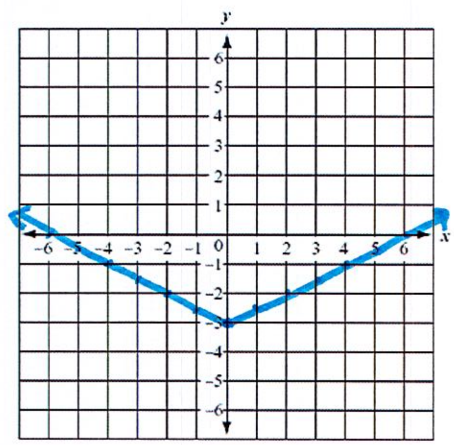
7. Graph $f(x)=(x+2)^2-4$



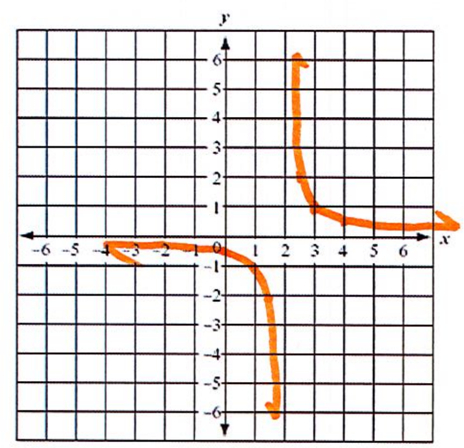
8. Graph $f(x)=\sqrt{x-3}$



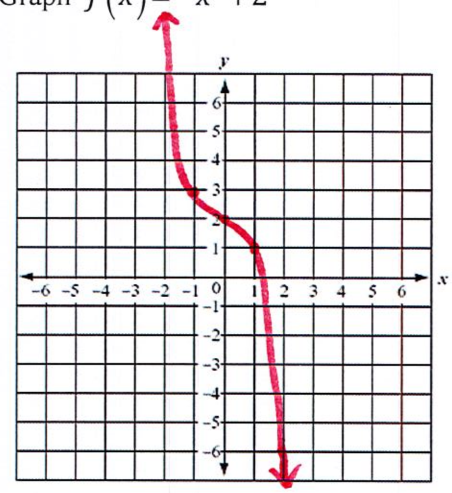
9. Graph $f(x)=\frac{1}{2}|x|-3$



10. Graph $f(x)=\frac{1}{x-2}$



11. Graph $f(x)=-x^3+2$



12. Graph $f(x)=(2x)^{\frac{1}{3}}$